**10-3 Ellipses Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**The center, vertices and foci of an ellipse**

1. Use the sliders in the Gizmotm to set *a* = 5 and *b* = 3. (To quickly set a slider to a specific number, type the number into the field to the right of the slider, and then press **ENTER.**)
   1. Vary the value of *h*. How does the graph change as the value of *h* changes?

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* 1. Vary the value of *k*. How does the graph change as the value of *k* changes?

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* 1. The red point on the graph represents the *center* of the ellipse. How do the coordinates of the center relate to the values of *h* and *k*?

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1. If the center of an ellipse his located at (4, −2), what are the numerators of its equation? Use the Gizmo to check your answer by dragging the center of the ellipse to (4, −2).

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1. Set *h* and *k* to 0. Vary the values of *a* and *b*.
   1. How does the shape of the ellipse change when these terms are changed?

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* 1. As you vary the values of *a* and *b*? What do you notice about the labels of these two sliders?

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* 1. How does the value of *a* compare to the value of *b*? As you vary the sliders, does this relationship ever change?

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* 1. When is the standard form of the equation of an ellipse Need Alt+ Need Alt= 1 ?

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* 1. When is the standard form of the equation of an ellipse Need Alt+ Need Alt= 1 ?

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1. The blue points on the graph represent the *vertices* of the ellipse.
   1. What is the relationship between the positions of the vertices and the values of *a* and *b*?

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* 1. Try manipulating the vertices directly by dragging them on the graph. How does the equation of the ellipse change as the position of the vertices changes?

1. The green points on the graph represent the *foci* of the ellipse. Drag the foci towards and away from the center of the ellipse.
   1. How does the shape of the ellipse change?

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* 1. What happens to the ellipse if you drag the foci onto the center?

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1. The *major axis* of an ellipse is the line segment that passes through the center and foci, and connects two of the vertices. The *minor axis* runs perpendicular to the major axis and connects the other two vertices. In the Gizmo, create an ellipse whose major axis is horizontal. Then, modify it so that its major axis is vertical. How does the standard form of the equation of the ellipse change when the orientation of the major axis changes?

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**The geometric definition of the ellipse**

The geometric definition of an ellipse is the set of all points in the plane such that the sum of the distances from two fixed points (the foci) is constant.

1. Turn on **Explore geometric definition.** A purple point will appear on the ellipse, along with two line segments labeled L1 and L2. Drag the purple point around the ellipse.
   1. Do the lengths of L1 and L2 change? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
   2. Does the sum of the lengths change? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
   3. How do these observations relate to the geometric definition of the ellipse.

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**The Pythagorean relationship**

The Pythagorean relationship is a direct consequence of the geometric definition of an ellipse, and describes how the foci and vertices are related.

1. Make sure **Explore geometric definition** is turned on. Set *a* = 6, *b* = 4, *h* = 0 and *k* = 0. Drag the purple point so that it is directly above the center of the ellipse. The point is directly above the center when L1 is equal to L2.
   1. Notice that two triangles are formed between the two foci, one on each side of the *y*-axis. What is the height of these triangles? Which term in the equation of the ellipse does the height correspond to: *a*, *b*, *h* or *k*?

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* 1. The line segments L1 and L2 correspond to the hypotenuses of the triangles. What is the length of each hypotenuse? Which term in the ellipse's equation does this correspond to?

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* 1. Turn off **Explore geometric definition,** and turn on **Show Pythagorean relationship.** How can you use this relationship to find the distance between the center of the ellipse and one of its foci?

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* 1. How does the Pythagorean relationship change if the ellipse is taller than it is wide?

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1. Determine the distance between the center of an ellipse and either of its foci, when *a* = 5, *b* = 4, *h* = 2 and *k* = −2? Check your answer by graphing the ellipse in the Gizmo.